Permutations and Combinations Quiz – Part 1 (Take-Home) 26 **Name:**______________________

1. In a particular jurisdiction, license plates consist of any **three non-repeating letters** (where the first of which is not the letters "I" or "O"), **followed by the digit** 6,7, or 8, **followed by any two digits**. (Repetition of digits is ok!). Determine the total number of possible license plates.

$$\underbrace{\underset{\text{Not "I" or "O"}}{24} * \underset{\text{Can't be 1st \#}}{25} * \underset{\text{Can't be 1st or 2nd}}{24}}_{\text{LETTERS (26)}} * \underbrace{\underset{\text{6, 7, or 8}}{3} * \underbrace{10 * 10}_{\text{Any two digits}}}_{\text{DIGITS (10)}} \quad \boxed{= 4\,320\,000}$$

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2. Consider the word "SASKATCHEWAN". How many ways are there to…

(a) Arrange all of the letters in the word

$\dfrac{12!}{2!\,3!}$ ← # of letters ← divide repetition (2 S's, 2 A's)

$\boxed{=39\,916\,800}$

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(b) Arrange all of the letters in the word – if the first two letters must be "KW" *(in that order)*

Arrange remaining letters

$\underset{K}{1} \times \underset{W}{1} \times \underbrace{\dfrac{10!}{2!\,3!}}_{\text{"SASATCHEAN"}} = \boxed{302\,400}$

(c) Arrange all of the letters in the word – if all of the S's must be kept together.

$\boxed{SS}, \boxed{A}, \boxed{K}, \boxed{A}, \boxed{T}, \boxed{C}, \boxed{H}, \boxed{E}, \boxed{W}, \boxed{A}, \boxed{N}$

count as one letter

$\dfrac{11!}{3!} = \boxed{6652800}$ ← # of letters ← A's

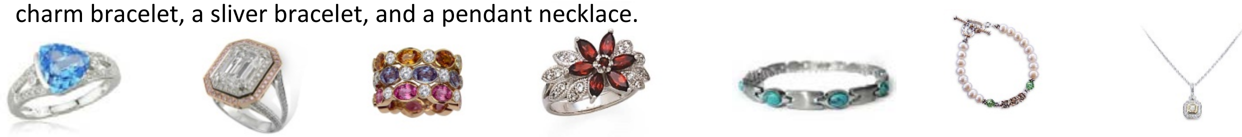
* only one way to arrange S's among themselves

(d) Arrange all of the letters in the word – if it must begin with exactly two A's.

$\underset{A}{1} \times \underset{A}{1} \times \underset{\text{NOT A}}{9} \times \dfrac{9!}{2!}$ ← Remaining letters ← S's

$= \boxed{1\,632\,960}$

3. An advertising executive is designing an ad for a jewelry company. The company has several distinct items it wishes to display in a single row across the bottom of the page; a diamond ring, a rhinestone ring, a sapphire ring, a floral ring, a charm bracelet, a sliver bracelet, and a pendant necklace.



How many ways can these items be displayed if:

(a) The rings must together on the left side of the row, and the bracelets must be together on the right side.

$\underset{R}{4} \cdot \underset{R}{3} \cdot \underset{R}{2} \cdot \underset{R}{1} \cdot \underset{N}{1} \cdot \underset{B}{2} \cdot \underset{B}{1} = \boxed{48}$

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(b) All of the rings and all of the bracelets must be together.

count as ONE item

$\boxed{RRRR}, \boxed{BB}, \boxed{N}$ ← 3 items to arrange

$= 3! \times 4! \times 2! = \boxed{288}$

Arrange rings, bracelets among themselves

(c) Only the rings must be together.

$\boxed{RRRR}, \boxed{B}, \boxed{B}, \boxed{N}$

Arrange 4 items.... (Counting Rings as ONE)

$4! \times 4! = \boxed{576}$

(d) The rings must alternate with the other pieces.

$\underset{R}{4} \cdot \underset{\text{other}}{3} \cdot \underset{R}{3} \cdot \underset{O}{2} \cdot \underset{R}{2} \cdot \underset{O}{1} \cdot \underset{R}{1}$

$= \boxed{144}$

4. Kate is organizing her linen closet. She has 8 identical towels; five are white, one is green, and two are blue. Determine the number of way she can stack the towels, one on top of the other in each scenario:

(a) If the top and bottom towel must be white.

Locked In (Don't Consider)

⇒ "$\underline{W}$ $\underbrace{WWWGBB}$ $\underline{W}$"

Arrange middle 6 towels....

Divide Repetition

$= \dfrac{6!}{3!\,2!} \Rightarrow = \boxed{60}$

Whites Blue

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(b) If the blue towels are not to be together.

W, W, W, W, W, G, BB

Count Blue's as ONE

whites $\dfrac{7!}{5!} = \boxed{42}$

5. In the NHL's Central Division, there are seven teams, the top three of which are guaranteed to make the playoffs. Determine the number of total possible groups of three teams that could make the playoffs, if the Winnipeg Jets are guaranteed to not make the playoffs.

"Choose remaining 2 playoff teams, from the 6 NON-JETS teams!"

$_6C_2 = \boxed{15}$

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(order doesn't matter, just want # of groups)

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6. At the conclusion of a sales seminar, all of the attendees shook hands – with a total of 105 handshakes taking place. Assuming each attendee shook the hand of each other attendee, and no attendees shook hands more than once, set up and algebraically solve an equation to determine the number of attendees there were.

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$$_nC_2 = 105$$

$$\frac{n!}{(n-2)!\,2!} = 105$$

$$\frac{n\cdot(n-1)\cdot\cancel{(n-2)!}}{\cancel{(n-2)!}} = 105 \times 2$$

$$n^2 - n - 210 = 0$$

$$(n-15)(n+14) = 0$$

$\boxed{n = 15}$ or $\cancel{-14}$ extraneous

7. Evaluate without a calculator *(Show all steps/ reasoning)*: $_{50}C_{48}$

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$$\frac{50!}{(50-48)!\,48!} \Rightarrow \frac{50!}{2!\,48!} \Rightarrow \frac{50\cdot 49\cdot \cancel{48!}}{2\cdot \cancel{48!}} \Rightarrow \boxed{1225}$$

8. From a group of 8 girls and 6 boys, a student council committee of five members must be formed. How many ways can this committee be formed if:

(a) There are no restrictions

$_{14}C_5 = \boxed{2002}$

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(b) One of the girls, Penelope, must be in the committee

Still need 4 more, from 13 other students → $_{13}C_4 = \boxed{715}$

(c) The committee must consist of at least one girl. (Disregard the restriction described in (b) above)

CASE 1	CASE 2	CASE 3	CASE 4	CASE 5
1G, 4 B's	2G, 3B	3G, 2B	4G, 1B	5G, No B's
$_8C_1\cdot{}_6C_4$	$_8C_2\cdot{}_6C_3$	$_8C_3\cdot{}_6C_2$	$_8C_4\cdot{}_6C_1$	$_8C_5\cdot{}_6C_0$

$= 120 + 560 + 840 + 420 + 56 \Rightarrow = \boxed{1996}$

METHOD 2

Total Poss. Groups of 5 − Groups w/ NO Girls (Choose only Boys)

$_{14}C_5 - {}_6C_5\cdot{}_8C_0$

$\boxed{= 1996}$ ✓

9. A wrestling coach has 8 wrestlers, and from this group must select three to attend city finals. How many ways can this be done if two of the boys – Omar and Sven – cannot both go.

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Total Possible − Groups w/ Both Omar & Sven $= \boxed{50}$

$_8C_3 - {}_6C_1$ ← Need 1 more to go w/ Omar & Sven

10. From a standard deck of 52 cards, a five-card poker hand is dealt. Determine the number of hands that would consist of:

(a) Two kings, two jacks, and an ace

$\underbrace{_4C_2}_{\text{kings}}\cdot\underbrace{_4C_2}_{\text{Jacks}}\cdot\underbrace{_4C_1}_{\text{Ace}} = \boxed{144}$

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(b) Three-of-a-kind - Three kings

$_4C_3\cdot{}_{48}C_2 = \boxed{4512}$

↖ Need 2 other cards from the 48 "NON-kings"

(c) Any four-of-a-kind

$_4C_4\cdot{}_{48}C_1\cdot{}_{13}C_1 = \boxed{2496}$

↑ All 4 of that card ↑ Any other card ↖ Which Rank of card? (2, 3, k, Ace, etc...)

11. From the letters in the word "TOFIELD" determine:

(a) The number of 4-letter arrangements that can be made, consisting of "*vowel, vowel, consonant, consonant*". (in that order)

$\underset{V}{3}\cdot\underset{V}{2}\cdot\underset{C}{4}\cdot\underset{C}{3} = \boxed{72}$

(b) The number of ways any two vowels and two consonants can be selected. (not arranged!)

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$3\cdot 2\cdot{}_4C_2 = \boxed{18}$

(c) The number of ways each of the groups of four letters (from part B) can be arranged.

$4\cdot 3\cdot 2\cdot 1 = \boxed{24}$

(d) Use your result from part (c) to determine the number of possible four-letter arrangements that can be made, consisting of two vowels and two consonants.

$18 \times 24 = \boxed{432}$

For each of the 18 Groups of "2 vowels, 2 Const.", they can be arranged 24 ways...

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